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DIVISIBILITY TEST FOR A NUMBER 37

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Abstract. The number 37 is the first two-digit prime number where both digits 3 and 7 are odd primes. A special property of 37 is that digits repeat when it is multiplied by 3, 6, 9, 12, 15, 18, 21, 24, and 27 (e.g., $37 \times 3 = 111$, $37 \times 27 = 999$). This paper establishes rigorous criteria for determining if an integer is divisible by 37 without actual division, leveraging properties of congruences and modular arithmetic.

♣ Note to author: Use 2000 Mathematics Subject Classification.

1. Introduction

The integer 37 stands out as a unique arithmetical curiosity, being the first two-digit prime formed exclusively by prime digits[cite: 9, 10].Its most celebrated property is its relationship with multiples of 3, resulting in "repdigits"[cite: 11]. For example:

$$37 \times 3 = 111$$
$$37 \times 6 = 222$$
$$..$$
$$..$$
$$..$$
$$37 \times 27 = 999$$

The primary objective of this paper is to establish a systematic test for divisibility by 37 for integers of arbitrary length using modular arithmetic.

1.1. Integer Representation. Any positive integer N with $k+1$ digits can be expressed in base 10 as:

$$N = a_k 10^k + a_{k-1} 10^{k-1} + \cdots + a_1 10^1 + a_0 10^0$$

1.2. Fundamental Connection: Powers of 10 and 37. Since $37 \times 27 = 999$, it follows that $1000 - 1 = 999$, which implies:

$$10^3 \equiv 1 \pmod{37})]$$

This suggests that divisibility by 37 is linked to grouping digits into blocks of three[cite: 48].

Key words and phrases. Divisibility test, congruences, prime number, Natural numbers.
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2. Theorems and Proofs

Theorem 1. $(10^{3n} + 36)$ is divisible by 37 for $n \in \mathbb{N}$.

Proof. We know $10^3 \equiv 1 \pmod{37}$.

$$\begin{aligned} 10^{3n} + 36 &= (10^3)^n + 36 \\ &\equiv 1^n + 36 \pmod{37} \\ &\equiv 1 + 36 = 37 \equiv 0 \pmod{37} \end{aligned}$$

3. Background and Preliminaries

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Thus, $(10^{3n} + 36)$ is divisible by 37. □

Theorem 2. $(10^{3n-1} + 11)$ is divisible by 37 for $n \in \mathbb{N}$.

Proof. $10^3 \equiv 1 \pmod{37}$.

$$\begin{aligned} 10^{3n-1} + 11 &= \frac{10^{3n}}{10} + 11 \\ &\equiv \frac{1}{10} + 11 \pmod{37} \\ &\equiv \frac{1 + 110}{10} = \frac{111}{10} \pmod{37} \end{aligned}$$

Since 111 is divisible by 37 ($37 \times 3 = 111$), the expression is divisible by 37 □

Theorem 3. $(10^{3n+1} + 27)$ is divisible by 37 for $n \in \mathbb{N}$.

Proof. $10^3 \equiv 1 \pmod{37}$.

$$\begin{aligned} 10^{3n+1} + 27 &= (10^3)^n \cdot 10 + 27 \\ &\equiv 1 \cdot 10 + 27 = 37 \equiv 0 \pmod{37} \end{aligned}$$

Thus, the statement is true. □

5. Divisibility Criterion

An integer N is divisible by 37 if and only if the sum of its three-digit blocks (S), partitioned from right to left, is divisible by 37.

$$N \equiv \sum C_i \pmod{37}$$

6. Practical Examples

- **Example 1:** $4847 \to 4 + 847 = 851$. Since $851 = 37 \times 23$,so 4857 is divisible by 37.
- **Example 2:** $15,673,681 \to 15 + 673 + 681 = 1369$. Then $1 + 369 = 370$, which is divisible by 37.

7. Mathematical Properties of 37

Property	**Value**	**Significance**
Prime Type	12th Prime	Fundamental building block].
Digit Relation	Permutable	37 and 73 are both prime.
Decimal Period	3-digit repeating	$1/37 = 0.\overline{027}$.
Repdigit Basis	$37 \times 3 = 111$	Foundation of the rule.

8. Conclusion

This study validates a computationally efficient criterion for divisibility by 37. By grouping digits into blocks of three, the complex division of a large number N is transformed into a simpler sum S where $N \equiv S \pmod{37}$.

References

[1] David M. Burton, *Elementary Number Theory*, 6th Edition, Tata McGraw-Hill.
[2] S. Axer, *Undergraduate Texts in Mathematics*, Springer International Edition, 2002.

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