Ganitam: Frontier Journal of Computational and Applied Mathematics
Vol. 1 No. 1 (June, 2026)

Received: 19th December 2025
Revised: 18th March 2026
Accepted: 20th March 2026

RELATIONSHIP BETWEEN THE PRIME DIVISORS AND RECURRING DIGITS IN THE QUOTIENT

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Abstract. This research paper proposes some results that are related to a composite numbers have unit digit 1 and its corresponding odd prime divisor[cite: 5]. Eventually the recurring digit occur in the composite number and its corresponding quotient are related to the number of recurring digits in the composite number[cite: 6].

Keywords: Prime number, composite numbers, recurring digit, natural number[cite: 8].

1. INTRODUCTION

Properties of prime number and composite numbers have been studied since 200 years[cite: 9]. Use of prime numbers in cryptography, in coding theory was virtually unheard before 100 years[cite: 10]. Now in this paper, I have found the relationship between the composite number having unit digit 1 and its corresponding prime divisor[cite: 11]. The Two-digit prime numbers are 11, 13, 17 and 19 have a special property in NUMBER THEORY[cite: 12].

Definitions.

- **Prime Number:** A Prime number is a whole number greater than 1, that is divisible by 1 and itself only[cite: 14]. For example: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37 etc. up to infinity[cite: 15].
- **Composite Number:** A number which is not prime number is called composite number[cite: 16]. For example: 4, 6, 8, 9, 10, 12, 15... etc.[cite: 17].

Key words and phrases. Divisibility test, congruences, prime number, Natural numbers.

* This research is supported by the Research Students .

2. MAIN RESULTS

TYPE-I. 11 is a first two digit prime number. Let us consider the composite number having unit digit 1 say 121[cite: 21].

$$
\begin{gathered}
121 \div 11 = 11 \\
1,221 \div 11 = 111 \\
12,221 \div 11 = 1,111 \\
122,221 \div 11 = 11,111 \\
1,222,221 \div 11 = 111,111 \\
.. \\
.. \\
.. \\
.. \\
\infty
\end{gathered}
$$

In all the above infinite equations we can observe easily that if digit 2 is recurring n times in the above composite numbers, then in the corresponding quotients the digit 1 is recurring n times for a natural number n[cite: 29].

TYPE-II. 13 is a prime number. Let us consider the composite number having unit digit 1 say 221[cite: 32, 33].

$$
\begin{gathered}
221 \div 13 = 17 \\
2,301 \div 13 = 177 \\
23,101 \div 13 = 1,777 \\
231,101 \div 13 = 17,777 \\
2,311,101 \div 13 = 177,777 \\
.. \\
.. \\
.. \\
\infty
\end{gathered}
$$

In all the above infinite equations we can observe easily that if the digit 1 is recurring n times in the above composite numbers, then in the corresponding quotients the digit 7 is recurring $(n+1)$ times for a natural number n[cite: 42].

TYPE-III. 17 is a prime number. Let us consider the composite number having unit digit 1, say 221[cite: 45].

$$221 \div 17 = 13$$
$$2,261 \div 17 = 133$$
$$22,661 \div 17 = 1,333$$
$$226,661 \div 17 = 13,333$$
$$2,266,661 \div 17 = 133,333$$
$$..$$
$$..$$
$$..$$
$$\infty$$

In all the above infinite equations we can observe easily that if the digit 6 is recurring n times in the above composite number, then in the corresponding quotients the digits 3 is recurring n times for a natural number n[cite: 52].

TYPE-IV. 19 is a prime number. Let us consider the composite number having unit digit 1, say 361[cite: 56, 57].

$$361 \div 19 = 19$$
$$3,781 \div 19 = 199$$
$$37,981 \div 19 = 1,999$$
$$379,981 \div 19 = 19,999$$
$$3,799,981 \div 19 = 199,999$$
$$..$$
$$..$$
$$..$$
$$\infty$$

In all the above infinite equations we can observe easily that if the digit 9 is recurring n times in above composite numbers, then in the corresponding quotients the digit 9 is recurring $(n+1)$ times, for a natural number n[cite: 65].

3. CONCLUSION

For other prime divisors say 23, 29, 31, 37 the quotients will be repeating exactly in the same pattern[cite: 69].

- **Note 1 (Prime 31):** Recurring digits follow TYPE-I (Prime divisor 11)[cite: 81].
- **Note 2 (Prime 23):** $391 \div 23 = 17$, $4,071 \div 23 = 177$, $40,871 \div 23 = 1,777$. These follow TYPE-II patterns[cite: 86, 87, 88, 92].
- **Note 3 (Prime 37):** $481 \div 37 = 13$, $4,921 \div 37 = 133$. These follow TYPE-III patterns[cite: 97, 99, 106].
- **Note 4 (Prime 29):** $551 \div 29 = 19$, $5,771 \div 29 = 199$. These follow TYPE-IV patterns[cite: 111, 112, 118].

4. APPLICATIONS

(1) Cryptography and coding theory[cite: 124].
(2) Identifying patterns of digits in numbers[cite: 126].
(3) Simplifying calculations in finance and engineering[cite: 128].
(4) Creation of magic numbers[cite: 130].
(5) Teaching patterns in modular arithmetic or prime numbers[cite: 132].

References

[1] David M. Burton, *Elementary Number Theory*, Tata McGraw-Hill Publishing Company Limited[cite: 134, 135].
[2] John Stillwell, *Undergraduate Texts in Mathematics*, Springer International Edition[cite: 136, 137, 138].

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