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AN ACCURATE SOLUTION OF NONLINEAR VOLTERRA INTEGRAL EQUATIONS USING HOSOYA POLYNOMIAL

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ABSTRACT: This paper aims to solve nonlinear Volterra integral equations by the Hosoya polynomial method (HPM). This method is implemented by one of the graph theory concepts called Hosoya polynomial. The algorithm of HPM expands the required solution as a set of containing continuous polynomials over the interval [0, 1]. The efficiency of this method is revealed by considering numerical examples and obtained results are comparing favorably with corresponding accurate solutions and errors. This work is evidence that graph polynomials can be utilized to solve numerical problems.

Keywords: Path of a graph, Volterra integral equations, Hosoya polynomial.

Mathematics Subject Classification: 65D30, 45B05, 05C12.

1Introduction

Integral equations have plenty of applications in numerous areas of science and technology, still have been studied drastically each on the platform of theoretic and practical senses. In unique, critical equations get up in fluid mechanics, kinetics in chemistry, organic fashions, stable country physics and many other fields. Analytically, it is hard to solve these equations in most of the cases. There are various methods to settle integral equations such as the Adomian decomposition technique, successive substitutions, Laplace transformation method, Picard's method [1] etc.

In this paper a Hosoya polynomial method is developed for the numerical solution of the nonlinear Volterra integralequation,

$$\omega(x) = \tau(x) + \int_0^x k(x,t)\,[y(t)]^m\,dt, \qquad 0 \le x, t \le 1, \tag{1}$$

Let us assume that $y(x)$ is a unique solution of Eqn. (1) which has to be determined. Where $[y(t)]^m$, $m \ge 1$ is the nonlinear term, $f(x)$ and the kernel $k(x,t)$ are assumed to bein $L^2(R)$ over the interval $0 \le x, t \le 1$. Numerous methods are available in the literature concerning the numericalsolutions of nonlinear Volterra integral equation such as chebyshev method [2], single-term walsh series method [3], sinc function [4], optimal homotopy asymptotic method [5], block-pulse functions and taylor series [6, 7] and wavelet method [8].

Hosoya polynomials have been recently used for the solution differential equations and integral equations. The numerical solution of the Fredholm integral equation by Hosoya polynomial has been introduced [25], whereas in [26] a comparative study on numerical solution of Fredholm integral equation by Haar wavelet method and Hosoya polynomial method has been carried out. Applied Hosoya polynomial method for the numerical solution of delay differential equations [27].

In this paper, we applied the Hosoya polynomial method to solve different nonlinear Volterra integral equation, utilizing Hosoya polynomials of paths as a basis. The technique to solve the nonlinear Volterra integral equation is to reduce into an system of algebraic equations, which can be efficiently solved using MATLAB solver. Illustrative numerical problems are considered to demonstrate the efficiency of the Hosoya polynomial method and those results are comparing favorably with the corresponding exact solutions and errors. Also, this method gives a more accurate solution than the existing methods.

The paper is structured as follows: In section 2, some basic definitions and properties of the Hosoya polynomial of graphs are included. Section 3, represents a description of the Hosoya polynomial method. In section 4, we solve some illustrative examples and demonstrate the numerical results with high accuracy and efficiency of HPM. The conclusion of the proposed method is given in section 5.

2 Hosoya Polynomial

Throughout this work, we consider simple graph G and let *V* be nonempty finite set has *n* vertices of graph G. Let X be a set with *m* number of unordered pairs of distinct vertices of set *V*. Thus, every pair, consider $x=(u,v)$ of points in X is an edge joined by points *v* and *u* which are adjacent to each other. Consider $v_1, v_2, \ldots, v_n$ are the vertices of graph a G. Let P_n be a path graph has *n* vertices $v_1, v_2, \ldots, v_n$, where v_i and v_{i+1}, $1 \le i < n$ are adjacent to each other. The number of edges present in path P_n is the length of P_n. In a connected graph G [13], each pair of points are connected by some path. The *distance* between such vertices v_i and v_j in G is same as the length of the shortest path which joins vertices v_i and v_j. That is given by $d(u_i, v_j)$.

Harold Wiener [21] introduced the *Wiener index* of graph G, as the sum of the distances of these pairs of G. Let *WI*(G) be a *Wiener index* of a connected graph G, that is,

$$WI(G) = \sum_{1 \le i < j \le n} d(u_{i,} v_j).$$

In 1988, the study of Hosoya polynomial [14] provides necessary content about distance based graph invariants and also pointed out that Hosoya polynomials connection with Wiener index is elementary. Here $H(G,\lambda)$ denotes *Hosoya polynomial* and is defined as,

$$H(G,\lambda) = \sum_{k \ge 0} d(G,k)\lambda^k \tag{2}$$

where λ is the parameter, and $d(G,k)$ is the number of pairs of vertices of graph G that are at distance *k*. The relation between the Wiener index $WI(G)$ and the Hosoya polynomial $H(G,\lambda)$, is reported in [14, 16]:

$$WI(G) = H'(G,1),$$

where $H'(G,\lambda)$ indicates first derivative of $H(G,\lambda)$.

Many authors studied these concepts like Hosoya polynomial of tress [19, 20], tori [10], zigzag polyhexnanotorus[11], zig-zag open-ended nanotubes [24], benzenoid graphs [12, 23], Fibonacci and Lucas cubes [15], composite graphs [18], armchair open-ended nanotubes [22], etc.

The paths P_4, P_3 and P_2 are shown in *Fig. 1*.

Fig. 1. P_4, P_3 and P_2 .

The Hosoya polynomial of a path P_n is given as

$$H(P_n,\lambda) = n + (n-1)\lambda + (n-2)\lambda^2 + \ldots + [n-(n-2)]\lambda^{n-2} + [n-(n-1)]\lambda^{n-1}$$
.

Particularly,

$$H(P_4,\lambda) = \lambda^3 + 2\lambda^2 + 3\lambda + 4,\ H(P_3,\lambda) = \lambda^2 + 2\lambda + 3,\ H(P_2,\lambda) = \lambda + 2.$$

Function approximation:

A function $\tau(x) \in L^2[0,1]$ is expanded as:

$$\tau(x) = \sum_{i=1}^{n} a_i H(P_i, x) = A^T H_P(x) \tag{3}$$

where $H_P(x)$ and A are $n \times 1$ matrices given by:

$$A = [a_1, a_2, \ldots, a_n]^T, \tag{4}$$

and

$$H_P(x) = [H(P_1, x), H(P_2, x), \ldots, H(P_n, x),]^T. \tag{5}$$

Hosoya polynomial Matrix

We can generalize the Hosoya polynomial of a matrix using the collocation points as follows,

$$H_n(x) = \begin{cases} H_1(x_i) = 1 \\ H_2(x_i) = x_i + 2 \\ H_3(x_i) = x_i^2 + 2x_i + 3 \\ \vdots \\ H_n(x_i) = n + (n-1)x_i + (n-2)x_i^2 + \ldots + [n-(n-2)]x_i^{n-2} + [n-(n-1)]x_i^{n-1} \end{cases}$$

where $x_i = \dfrac{i-0.5}{n}$, $i = 1,2,\ldots,n.$

Suppose, for n = 2

$$H_2(x) = \begin{cases} H_1(x_i) = 1 \\ H_2(x_i) = x_i + 2 \end{cases}, \quad \text{for } x_i = \frac{i-0.5}{n},\ i = 1,2$$

The matrix 2×2 is of the form

$$H_{2\times 2} = \begin{bmatrix} 1 & 1 \\ 2.25 & 2.75 \end{bmatrix}$$

Similarly, for n = 4

$$H_4(x) = \begin{cases} H_1(x_i) = 1 \\ H_2(x_i) = x + 2 \\ H_3(x_i) = x_i^2 + 2x_i + 3 \\ H_4(x_i) = x_i^3 + 2x_i^2 + 3x_i + 4 \end{cases}, \text{ for } x_i = \frac{i-0.5}{n},\ i = 1,2,3,4$$

The matrix 4×4 is of the form

$$H_{4\times 4} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2.125 & 2.375 & 2.625 & 2.875 \\ 3.265 & 3.890 & 4.640 & 5.515 \\ 4.408 & 5.459 & 6.900 & 8.826 \end{bmatrix}$$

Similarly, for n = 8, we get the 8x8 matrix of the form

$$H_{8\times8}=\left[\begin{array}{cccccccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\
2.0625 & 2.1875 & 2.3125 & 2.4375 & 2.5625 & 2.6875 & 2.8125 & 2.9375\\
3.1289 & 3.4101 & 3.7226 & 4.0664 & 4.4414 & 4.8476 & 5.2851 & 5.7539\\
4.1955 & 4.6394 & 5.1633 & 5.7790 & 6.4982 & 7.3327 & 8.2941 & 9.3942\\
5.2622 & 5.8698 & 6.6135 & 7.5283 & 8.6552 & 10.0412 & 11.7390 & 13.8071\\
6.3288 & 7.1006 & 8.0667 & 9.2936 & 10.8686 & 12.9033 & 15.5379 & 18.9442\\
7.3955 & 8.3313 & 9.5208 & 11.0659 & 13.1135 & 15.8710 & 19.6245 & 24.7601\\
8.4622 & 9.5621 & 10.9752 & 12.8413 & 15.3763 & 18.9113 & 23.9449 & 31.2126
\end{array}\right]$$

Next, for n = 16 as,

$$H_{16\times16}=\left[\begin{array}{cccccccccccccccc}
1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0\\
2.0 & 2.1 & 2.2 & 2.2 & 2.3 & 2.3 & 2.4 & 2.5 & 2.5 & 2.6 & 2.7 & 2.7 & 2.8 & 2.8 & 2.9 & 3.0\\
3.1 & 3.2 & 3.3 & 3.5 & 3.6 & 3.8 & 4.0 & 4.2 & 4.3 & 4.5 & 4.7 & 5.0 & 5.2 & 5.4 & 5.6 & 5.9\\
4.1 & 4.3 & 4.5 & 4.8 & 5.0 & 5.3 & 5.6 & 5.9 & 6.3 & 6.7 & 7.1 & 7.6 & 8.0 & 8.6 & 9.1 & 9.7\\
5.1 & 5.4 & 5.7 & 6.0 & 6.4 & 6.8 & 7.3 & 7.8 & 8.4 & 9.0 & 9.7 & 10.4 & 11.3 & 12.2 & 13.3 & 14.4\\
6.2 & 6.5 & 6.9 & 7.3 & 7.8 & 8.3 & 9.0 & 9.7 & 10.4 & 11.3 & 12.3 & 13.5 & 14.8 & 16.3 & 18.0 & 19.9\\
7.2 & 7.6 & 8.1 & 8.6 & 9.2 & 9.9 & 10.6 & 11.5 & 12.5 & 13.7 & 15.1 & 16.7 & 18.6 & 20.8 & 23.3 & 26.3\\
8.2 & 8.7 & 9.3 & 9.9 & 10.6 & 11.4 & 12.3 & 13.4 & 14.7 & 16.2 & 17.9 & 20.0 & 22.5 & 25.5 & 29.1 & 33.5\\
9.3 & 9.8 & 10.4 & 11.2 & 12.0 & 12.9 & 14.0 & 15.3 & 16.8 & 18.6 & 20.8 & 23.4 & 26.6 & 30.5 & 35.4 & 41.4\\
10.3 & 10.9 & 11.6 & 12.4 & 13.4 & 14.4 & 15.7 & 17.2 & 18.9 & 21.0 & 23.6 & 26.8 & 30.8 & 35.8 & 42.1 & 50.2\\
11.3 & 12.0 & 12.8 & 13.7 & 14.8 & 16.0 & 17.4 & 19.0 & 21.1 & 23.5 & 26.5 & 30.3 & 35.0 & 41.2 & 49.1 & 59.6\\
12.4 & 13.1 & 14.0 & 15.0 & 16.2 & 17.5 & 19.1 & 20.9 & 23.2 & 25.9 & 29.4 & 33.8 & 39.4 & 46.7 & 56.5 & 69.7\\
13.4 & 14.2 & 15.2 & 16.3 & 17.5 & 19.0 & 20.7 & 22.8 & 25.3 & 28.4 & 32.3 & 37.3 & 43.8 & 52.4 & 64.2 & 80.5\\
14.4 & 15.3 & 16.4 & 17.6 & 18.9 & 20.5 & 22.4 & 24.7 & 27.4 & 30.9 & 35.2 & 40.8 & 48.2 & 58.2 & 72.2 & 92.0\\
15.5 & 16.4 & 17.6 & 18.8 & 20.3 & 22.1 & 24.1 & 26.6 & 29.6 & 33.3 & 38.1 & 44.3 & 52.6 & 64.1 & 80.4 & 104.2\\
16.5 & 17.5 & 18.7 & 20.1 & 21.7 & 23.6 & 25.8 & 28.5 & 31.7 & 35.8 & 41.0 & 47.8 & 57.1 & 70.1 & 88.9 & 116.9
\end{array}\right]$$

Kernel Matrix

The kernel matrix is the square matrix obtained by evaluating the kernel function k with Hosoya polynomial $H_P(x)$. As the number of samples n tends to infinity, certain properties of the kernel matrix show a convergent behavior.

The given kernel function as,

$$K = \int_0^x k(x,t)\left[\omega(t)\right]^m dt, \qquad 0 \le x,t \le 1,$$

Let us put n = 6 and m may be the any order, then by applying the collocation points

$$K_i = \int_0^x k(x_i,t)\,\omega(t)\,dt, \quad x_i = \frac{i-0.5}{6},\ i = 1,2,3,4,5,6$$

Then substitute the approximated truncated series ω(x) as,

$$\omega(x) = A^T H_P(x)$$

$$K_i = \int_0^x k(x_i,t)\left[A^T H_P(t)\right]^m dt,$$

Then reduce in the form of 6 x 6 matrix as, the kernel function is reduces in the form of 6 x 6 matrix,

$$K_i = \left[A^T\right]^m \left[\int_0^x k(x_i,t)\left[H_P(t)\right]^m dt\right],$$

$$\left[K\right]_{6\times6} = \left[A\right]_{1\times6}\left[(k)_{6\times6}(H)_{6\times6}\right],$$

In general,

$$\left[K\right]_{n\times n} = \left[A\right]_{1\times n}\left[(k)_{n\times n}(H)_{n\times n}\right]$$

which the given kernel function is reduces in the form of kernel matrix up to n x n matrices.

3 Description of Hosoya Polynomial Method (HPM)

Here, we consider the nonlinear Volterra integral equation,

$$\omega(x) = \tau(x) + \int_0^x k(x,t)[y(t)]^m dt, \qquad 0 \le x,t \le 1, \tag{6}$$

The steps to solve Eq. (6), are as follows:

Step 1 Using the above defined Eq. (6), approximate the truncated series as $\omega(x)$. That is,

$$\omega(x) = A^T H_P(x) \tag{7}$$

where $H_P(x)$ and A are defined in Eq. (5) and Eq. (4).

Step 2 Substitute Eq. (7) in Eq. (6), which results as,

$$A^T H_P(x) = \tau(x) + \int_0^x k(x,t)[A^T H_P(t)]^m \, dt, \tag{8}$$

Step 3 Next, we substitute the collocation point $x_i = \dfrac{i-0.5}{n}$, $i = 1,2,\ldots,n$ in Eq. (8), to obtain,

$$A^T H_P(x_i) = \tau(x_i) + \int_0^1 k(x_i,t)[A^T H_P(t)]^m \, dt, \tag{10}$$

$$A^T H_P(x_i) = \tau(x_i) + A^T \left[\int_0^1 k(x_i,t)[H_P(t)]^m \, dt \right]$$

where, $\tau(x) = [\tau(x_1), \tau(x_2), \tau(x_3), \ldots, \tau(x_n)]$ and

$$K = \int_0^x k(x_i,t)[H_p(t)]^m \, dt$$

Step 4 Thus, the integral equation converts into the system of algebraic equations with unknown coefficients as,

$$[A]_{1\times n}[H]_{n\times n} = [f]_{1\times n} + [A]_{1\times n}[(k)_{n\times n}(H)_{n\times n}]$$
$$[A]_{1\times n}[H_{n\times n} - k_{n\times n}H_{n\times n}] = [f]_{1\times n}$$
$$[A]_{1\times n}[H_{n\times n} - K_{n\times n}] = [f]_{1\times n}$$

Step 5 On solving this system of nonlinear algebraic equations, we obtain the Hosoya coefficients 'A' using the Matlab iterative method and then substitute these coefficients in Eq. (7). Hence, finally we claim the desired approximate solution of Eq. (6).

4 Numerical Experiment

To demonstrate the capability of this method, we consider a few illustrative examples from the literature and verify the accuracy and efficiency of the results:

$$E_{Max} = Error\ function = \|\omega_e(x_i) - \omega_a(x_i)\|_\infty = \sqrt{\sum_{i=1}^{n} [\omega_e(x_i) - \omega_a(x_i)]^2}$$

where, ω_e is exact solution and ω_a is approximate solution.Here, we consider for $m$up to the order two and three for the numerical solutions to compare with the exact solutions.

Example 1. Consider the nonlinear Volterra integral equation [2],

$$\omega(x) = 2x - \frac{x^4}{2} + \frac{1}{4}\int_0^x \omega^3(t)\,dt, \qquad 0 \le x < 1 \tag{10}$$

and the exact solution is $\omega(x) = 2x.$ Here we considered for $n = 3$ and $m = 3$, by the proposed technique to solve the Eq. (10) is reduced into system of algebraic equations. On solving this system, we get the three unknown Hosoya coefficients as,

$$a_1 = -4,\ a_2 = 2,\ a_3 = 0.$$

substituting these coefficients in $\omega(x) = A^T H_P(x)$ as,

$$\omega(x) = a_1(1) + a_2(x+2) + a_3(x^2+2x+3)\ ,$$

we obtain the accurate solution as same as exact solution $\omega(x) = 2x.$ Which is the required solution of Eqn. (10). This shows the efficiency of the present method.

Example 2. Consider nonlinear Volterra integral equations [3],

$$\omega(x) = \frac{-15}{56}x^8 + \frac{13}{14}x^7 - \frac{11}{10}x^6 + \frac{9}{20}x^5 + x^2 - x + \int_0^x (x+t)[\omega(t)]^3\,dt, \qquad 0 \le x \le 1, \tag{11}$$

and the exact solution is $\omega(x) = x^2 - x.$ On solving Eqn.(11) by the present method at $n = 3$ and $m = 3$, we get the coefficients $a_1 = 3,\ a_2 = -3,\ a_3 = 1.$ Substituting these in the corresponding expression of $\omega(x),$ we have the solution same as that of exact solution.

Example 3. Consider nonlinear Volterra integral equations [4],

$$\omega(x) = x - x^2 - \frac{x^5}{4} + \frac{2x^6}{5} - \frac{x^7}{6} + \int_0^x xt\,\omega^2(t)\,dt, \qquad 0 \le x \le 1, \tag{12}$$

which has an exact solution $\omega(x) = x - x^2.$ On solving Eqn.(12) by the present method at $n = 3$ and $m = 2$, we get the coefficients $a_1 = -3,\ a_2 = 3,\ a_3 = -1.$ Substituting these in the corresponding expression of $\omega(x),$ we have the solution same as that of exact solution. This shows the more accuracy and validity of the present technique.

Example 4. Consider nonlinear Volterra integral equations[5],

$$\omega(x) = x^2 + \frac{x^6}{12} - \frac{1}{2}\int_0^x t\,\omega^2(t)\,dt, \qquad 0 \le x \le 1, \tag{13}$$

which has an exact solution $\omega(x) = x^2$. On solving Eqn.(13) by the present method at n = 6 and m = 2, we get the coefficients $a_1 = 1,\ a_2 = -2,\ a_3 = 1,\ a_4 = 0,\ a_5 = 0,\ a_6 = 0.$ Substituting these in the corresponding expression of $\omega(x)$, we have the solution same as that of exact solution. This shows the more accuracy and validity of the present technique.

Example 5. Consider nonlinear Volterra integral equations [5],

$$\omega(x) = x - \frac{x^4}{4} + \int_0^x t\,\omega^2(t)\,dt, \qquad 0 \le x \le 1, \tag{14}$$

which has an exact solution $y(x) = x$. On solving Eqn.(14) by the present method at n = 6 and m = 2, we get the coefficients $a_1 = -2,\ a_2 = 1,\ a_3 = 0,\ a_4 = 0,\ a_5 = 0,\ a_6 = 0.$ Substituting these in the corresponding expression of $\omega(x)$, we have the solution same as that of exact solution. This shows the more accuracy and validity of the present technique.

Example 6. Consider nonlinear Volterra integral equations [5],

$$\omega(x) = x^2 + \frac{x^5}{10} - \frac{1}{2}\int_0^x \omega^2(t)\,dt, \qquad 0 \le x \le 1, \tag{15}$$

which has an exact solution $\omega(x) = x^2$. On solving Eqn.(15) by the present method at n = 6 and m = 2, we get the coefficients $a_1 = 1,\ a_2 = -2,\ a_3 = 1,\ a_4 = 0,\ a_5 = 0,\ a_6 = 0.$ Substituting these in the corresponding expression of $\omega(x)$, we have the solution same as that of exact solution. This shows the more accuracy and validity of the present technique.

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