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MATHEMATICAL FORMALISM OF THE VON BERTALANFFY EQUATION AND ITS APPLICATION TO PANCREATIC CANCER

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ABSTRACT. This paper presents a mathematical and computational study of pancreatic tumor growth based on the von Bertalanffy equation. Starting from biologically meaningful assumptions regarding nutrient absorption and metabolic loss, the governing equation is rigorously derived. The model is implemented in C++ and integrated into the Geant4 Monte Carlo framework to simulate a fractionated radiotherapy protocol. The methodology combines formal derivation, C++ implementation, and Monte Carlo simulation into a unified platform for investigating therapeutic responses in pancreatic cancer.

1. Introduction

Pancreatic cancer is one of the most aggressive solid malignancies and remains a major global health problem. According to the Global Cancer Observatory, there were 510,922 new pancreatic cancer cases and 467,409 deaths worldwide in 2022, confirming both the high incidence and severe lethality of this disease. Recent epidemiological reviews also indicate that the worldwide burden continues to increase, reinforcing the importance of quantitative tools capable of describing tumor progression [1, 2].

Among classical ordinary differential equation models for biological growth, the von Bertalanffy equation is a natural choice because its structure is compatible with nutrient-limited growth and metabolic balance [3, 4]. A focused keyword screening performed in Springer, PubMed, ScienceDirect, and SciELO recovered several application-oriented records, but no clearly retrieved paper centered on the explicit mathematical derivation of the von Bertalanffy law for pancreatic cancer. Therefore, the present work fills this gap by presenting the mathematical formalism of the von Bertalanffy equation and applying it to pancreatic cancer through numerical simulation [1].

1.1. General Objective. The general objective of this work is to present the mathematical formalism of the von Bertalanffy equation and apply it to the study of pancreatic cancer growth through Monte Carlo simulation in the Geant4 framework.

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1.2. Specific Objectives. The specific objectives are: first, to discuss the biological assumptions justifying the von Bertalanffy equation; second, to derive the governing equation rigorously; third, to implement the equation in C++ within Geant4; fourth, to reproduce a fractionated radiotherapy protocol computationally; and finally, to interpret the simulation results in the context of medical physics and oncology.

2. Methodology

This study integrates formal mathematical derivation, computational implementation in C++, and Monte Carlo simulation in Geant4. The overall methodological strategy is summarized in Table 1.

TABLE 1. Summary of methodological stages and resources.

Stage	Tool / Resource	Purpose
Derivation	Theoretical	Rigorous derivation of equation from biological assumptions.
Implementation	Geant4 (C++)	C++ implementation in Monte Carlo framework.
Comparison	Clinical study	Reproduction of radiotherapy protocol for comparison.

3. Mathematical Derivation and Theoretical Context

Ludwig von Bertalanffy proposed a differential equation based on metabolic balance, where growth results from competition between anabolic and catabolic processes. The original formulation is given by:

$$\frac{dW}{dt} = \eta S - kV, \quad (3.1)$$

where $W = W(t)$ represents biological mass, S is exchange surface, V is volume, $\eta > 0$ is the anabolic coefficient, and $k > 0$ is the catabolic coefficient.

Let the total variation of mass be governed by a balance law:

$$\frac{dW}{dt} = \lim_{\Delta t \rightarrow 0} \frac{W(t + \Delta t) - W(t)}{\Delta t}. \quad (3.2)$$

Assuming nutrient influx through boundary $\partial\Omega$:

$$\text{Anabolism} = \iint_{\partial\Omega} J_n dS = J_n S = \eta S. \quad (3.3)$$

Catabolic consumption occurs throughout volume Ω :

$$\text{Catabolism} = \iiint_{\Omega} \gamma dV = \gamma V = kV. \quad (3.4)$$

Substituting into the balance equation yields Equation (3.1). For a spherical tumor with radius $r(t)$:

$$V = \frac{4}{3}\pi r^3 \implies r = \left(\frac{3V}{4\pi}\right)^{1/3}. \quad (3.5)$$

Computing surface area S :

$$S = 4\pi r^2 = 4\pi \left(\frac{3V}{4\pi}\right)^{2/3} = (4\pi)^{1/3} 3^{2/3} V^{2/3} = cV^{2/3}, \quad (3.6)$$

where $c = (4\pi)^{1/3} 3^{2/3}$. Thus:

$$\frac{dW}{dt} = \eta c V^{2/3} - kW. \quad (3.7)$$

Assuming constant density ρ ($W = \rho V$):

$$\frac{dW}{dt} = \eta c \left(\frac{W}{\rho}\right)^{2/3} - k \left(\frac{W}{\rho}\right) = aW^{2/3} - bW, \quad (3.8)$$

where $a = \eta c \rho^{-2/3}$ and $b = k \rho^{-1}$.

Using the transformation $u = W^{1/3}$, we obtain:

$$3u^2 \frac{du}{dt} = au^2 - bu^3 \implies \frac{du}{dt} + \frac{b}{3}u = \frac{a}{3}. \quad (3.9)$$

Solving via integrating factor $\mu(t) = e^{(b/3)t}$ yields:

$$W(t) = \left(\frac{a}{b} + Ce^{-(b/3)t}\right)^3. \quad (3.10)$$

4. Results and Simulation

The clinical study reference and simulation parameters are compared in Table 2.

TABLE 2. Comparison between clinical protocol and simulation parameters.

Parameter	Clinical Data	Simulation Model
Treatment	Radiotherapy	Geant4 Monte Carlo
Sessions	6 sessions	6 simulated fractions
Duration	5 weeks	$t \in [0, 35]$ days
Dose	2 Gy	Clinical protocol
Reduction	30–50%	Predicted by Geant4
Initial Vol	Baseline	Reference initial state

The three-dimensional simulation of tumor regression across 6 radiotherapy fractions (7.5 Gy to 45.0 Gy total dose) was conducted using Geant4. The progression shows progressive volume reduction consistent with clinical findings.

5. Conclusion

This study successfully established an integrated framework combining formal mathematical derivation, C++ implementation, Monte Carlo simulation in Geant4, and clinical comparison. The proposed methodology provides both theoretical transparency and a visual, quantitative computational platform for examining radiotherapy responses in pancreatic cancer.

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